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Semi separation axioms in ideal closure spaces

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Abstract

In this paper, we initiate the concept of semi open sets in ideal closure spaces. In particular, we deliberate the properties of semi- open and semi-closed sets, union and intersection of semi open subsets in ideal closure spaces. Along with we make some views on semi separation axioms in ideal closure spaces.

Keywords: Semi-open, semi-closed, semi-T_0-space, semi-T_1-space, semi-T_2-spaces.
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1. Introduction

19th century onwards separation axioms plays an important role in topological space. The separation axioms are called T_0, T_1, T_2, which are introduced by various authors namely A.N. Kolmogorov, Frechet and Hausdorff. In topological space the concept of semi-open sets are introduced by N. Levine[9]. Later Maheswari and Prasad[10] are initiated the concept of semi separation axioms with semi-T_0, semi-T_1, semi-T_2 space using semi-open sets.

The separation properties in closure space were introduced by E.Cech[3]. Recently, R. Gowri and M.Pavithra introduced and discussed the properties of lower and higher separation axioms in ideal closure space [5], [6]. In this paper, semi separation axioms in ideal closure space are introduced and characterized using semi open sets. Along with some relationship between separation axioms and semi separation axioms in ideal closure space are derived.

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2. Preliminaries

In this section, we recall the basic definitions of ideal closure spaces.

Definition 2.1. [1] $(X, \mathfrak{I})$ be a topological space. An ideal I on a topological space is a collection of non empty collections of subsets of X which satisfies:

(1) $\emptyset \in I$,
(2) $A \in I, B \subseteq A$ implies $B \in I$,
(3) $A \in I, B \in I$ implies $A \cup B \in I$. If $(X, \mathfrak{I})$ is a topological space and I is an ideal on X. Then $(X, \mathfrak{I}, I)$ is called an ideal topological space or an ideal space.

Definition 2.2. [7] Let $P(X)$ be the power set of X. Then the operator
$(.)^* : P(X) \to P(X)$ is called a local function of A with respect to $\mathfrak{I}$ and I, is define as follows:For $A \subseteq X$, $A^*(I, \mathfrak{I}^*) = \{x \in X : U \cap A \notin I$ for every open set U containing x$\}$

Additionally, $cl^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology $\mathfrak{I}^*$ is finer than $\mathfrak{I}$.

Definition 2.3. [5] let (X,k) be a non-empty set. I be an Ideal on X.
Let $A^* : P(X) \to P(X)$ be a function of A with respect to I and $\mathfrak{I}$,.
Let $k^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology.
Then the function $k^* : P(X) \to P(X)$ satisfying,

(1) $k^*(\emptyset) = \emptyset$.
(2) $A \subseteq X \implies A \subset k^*(A)$
(3) $k^*(A \cup B)= k^*(A) \cup k^*(B)\ \forall A, B \subset X$
(4) $k^*(A) = k^*(k^*(A))\ \forall A \subset X$ is called a closure operator on X. The structure (X, I, k^*) is called an Ideal Closure Space.

Example 2.4. $X = \{a, b, c\}$ $\mathfrak{I} = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$. $I = \{\emptyset, \{c\}\}$

(1) $A = \{a, c\}$ $A^* = \{a, b\}$ $k^*(A) = A \cup A^* \implies k^*\{a, c\}$ =X.
(2) $A = \{b, c\}$ $A^* = \{b\}$ $k^*(A) = A \cup A^* \implies k^*\{b, c\} =\{b, c\}$.
(3) $A = \{a, b\}$ $A^* = \{a, b\}$ $k^*(A) = A \cup A^* \implies k^*\{a, b\} =\{a, b\}$.
(4) $A = X$ $A^* = \{a, b\}$ $k^*(A) = A \cup A^* \implies k^*(X) = X$
(5) $A = \emptyset$ $A^* = \emptyset$ $k^*(A) = A \cup A^* \implies k^*(\emptyset) =\emptyset$.
(6) $A = \{a\}$ $A^* = \{a, b\}$ $k^*(A) = A \cup A^* \implies k^*\{a\} =\{a, b\}$.
(7) $A = \{b\}$ $A^* = \{b\}$ $k^*(A) = A \cup A^* \implies k^*\{b\} =\{b\}$.
(8) $A = \{c\}$ $A^* = \emptyset$ $k^*(A) = A \cup A^* \implies k^*\{c\} =\{c\}$.

$k^*\{a, b\} =\{a, b\}; k^*\{b\} =\{b\}; k^*\{c\} =\{c\}; k^*\{a, b\} =\{a, b\}; k^*\{b, c\} =\{b, c\}; k^*\{c, a\} =X; k^*(X) =X.$ $k^*(\emptyset) =\emptyset$. Then (X, I, k^*) is an ideal closure space.

Definition 2.5. [5] A subset A of an ideal closure space (X, I, k^*) is said to be closed if $k^*(A) = A$.

Definition 2.6. [5] A subset A of an ideal closure space (X, I, k^*) is said to be open if $k^*(X - A) = X - A$ (i.e) $int^*(A) = A$.

Definition 2.7. [5] The set int A with respect to the closure operator k^* is defined as $int^*(A) = X - k^*(X - A)(i.e)[k^*(A^C)]^C$, where $A^C = X - A$.

Definition 2.8. [5] (X, I, k^*) is an ideal closure space than the associate topology on X is $\mathfrak{I}^* =\left\{A^C; k^*(A) = A\right\}$. Here $\mathfrak{I}$ is not equal to $\mathfrak{I}^*$

Definition 2.9. [5] A subset A in an ideal closure space (X, I, k^*) is called neighbourhood of x if $x \in int^*(A)$.

Definition 2.10. [5] Let (X, I, k^*) be an ideal closure space. An ideal closure space (Y, I, k^*_Y) is called a subspace of (X, I, k^*) if $Y \subseteq X$ and $k^*_Y(A) = k^*(A) \cap Y$, $\forall A \subseteq Y$.

Definition 2.11. [5] An ideal closure space (X, I, k^*) is said to be T_0 - space iff for every distinct points $x \neq y$ and $x \notin k^*(\{y\})$ or $y \notin k^*(\{x\})$.

Definition 2.12. [5] An ideal closure space (X, I, k^*) is said to be T_1-space iff for every distinct points $x \neq y$ and $x \notin k^*(\{y\})$ and $y \notin k^*(\{x\})$.

Definition 2.13. [5] An ideal closure space (X, I, k^*) is said to be Hausdorff or T_2-space if every distinct points $x \neq y$ and there exists disjoint open sets G and H such that $x \in G$ and $y \in H$.

3. Semi-Open and Semi-Closed Sets in Ideal Closure Space

In this section, we defined semi-open set and semi-closed set in ideal closure space and their basic properties such as union and intersection of semi open sets are investigated.

Definition 3.1. Let (X, I, k^*) be an ideal closure space then a subset A is said to be semi-open set if $A \subseteq k^*(int^*(A))$.

Definition 3.2. Let (X, I, k^*) be an ideal closure space then a subset A is said to be semi-closed set if $int^*(k^*(A)) \subseteq A$.

Theorem 3.3. A subset A in a ideal closure space (X, I, k^*) is semi-open if and only if $U \subset A \subset k^*(U)$ for some open set U.

Proof.
Sufficient Part:
Let $U \subset A \subset k^*(U)$. Then for $U = int^*(A)$, we have $int^*(A) \subset A \subset k^*(int^*(A))$ implies $A \subset k^*(int^*(A))$. Therefore A is semi-open.

Necessary Part:
Let A is semi-open then $A \subseteq k^*(int^*(A))$. Then $U \subset A$ for some open set U but $U \subset int^*(A)$ and thus $k^*(U) \subset k^*(int^*(A))$ then $A \subset k^*(U) \subset k^*(int^*(A))$. Hence $U \subset A \subset k^*(U)$

Theorem 3.4. Let A and B be subsets of (X, I, k^*) such that $B \subseteq A \subseteq k^*(B)$. If B is semi-open set then A is semi-open.

Proof. Let B be semi-open set. We have $B \subseteq k^*(int^*(B)) \subseteq k^*(int^*(A))$. Thus $k^*(B) \subseteq k^*(int^*(A))$ implies $A \subseteq k^*(B) \subseteq k^*(int^*(A))$. We get $A \subseteq k^*(int^*(A))$. Hence A is semi-open set.

Theorem 3.5. If A be open set then A is semi-open set in ideal closure space (X, I, k^*).

Proof. Let A is open set in (X, I, k^*). Then $A = int^*(A)$. Now $A \subset k^*(A) = k^*(int^*(A))$ implies $A \subset k^*(int^*(A))$. Hence A is semi-open in (X, I, k^*).

Remark 3.6. The following example shows that converse of the above theorem is not true.

Example 3.7. $X = \{a, b, c\}$, $\mathfrak{I} = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$, $I = \{\emptyset, \{a\}\}$.
(X, I, k^*) is followed by,
$k^*(a) = \{a\}$; $k^*(b) = \{a, b\}$; $k^*(c) = \{a, c\}$; $k^*\{a, b\} = \{a, b\}$; $k^*\{b, c\} = X$; $k^*\{c, a\} = \{a, c\}$; $k^*(X) = X$; $k^*(\emptyset) = \emptyset$.
Open sets are $X, \emptyset, \{b\}, \{c\}, \{b, c\}$.
Semi-open sets are $X, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{c, a\}$.
Here $\{a, b\}, \{a, c\}$ is semi-open but not open.

Result 3.8. *Let (X, I, k^*) be an ideal closure space and A be a semi-closed set in (X, I, k^*) then* $k^*(int^*(k^*(A))) \subseteq k^*(A)$.

Proof. Let (X, I, k^*) be an ideal closure space and let A be semi-closed set, Using the definition of semi closed set we have,
$int^*(k^*(A)) \subseteq A$,
Taking k^* operator on both sides,
$k^*[int^*(k^*(A))] \subseteq k^*(A)$.

Theorem 3.9. The union of semi-open set is semi-open in ideal closure space (X, I, k^*).

Proof. Let A and B be any semi-open sets in (X, I, k^*).
Now $A \subseteq k^*(int^*(A))$
$B \subseteq k^*(int^*(B))$
Then $A \cup B \subset k^*(int^*(A)) \cup k^*(int^*(B))$
$\Longrightarrow A \cup B \subset k^*((int^*(A)) \cup (int^*(B))$
$\Longrightarrow A \cup B \subset k^*(int^*(A \cup B))$.

Hence $A \cup B$ is semi-open sets.

Result 3.10. *The following example shows that the intersection of two semi-open sets need not to be a semi-open set.*

Example 3.11. $X = \{a, b, c\}$, $\Im = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$, $I = \{\emptyset, \{a\}\}$.
(X, I, k^*) is followed by,
$k^*(a) = \{a\}$; $k^*(b) = \{a, b\}$; $k^*(c) = \{a, c\}$; $k^*\{a, b\} = \{a, b\}$; $k^*\{b, c\} = X$; $k^*\{c, a\} = \{a, c\}$; $k^*(X) = X$; $k^*(\emptyset) = \emptyset$.
Open sets are $X, \emptyset, \{b\}, \{c\}, \{b, c\}$.
Semi-open sets are $X, \emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{c, a\}$.
Then take the semi-open sets, $U = \{a, b\}$, $V = \{c, a\}$. $U \cap V = \{a\}$, which is not a semi-open set.

Theorem 3.12. $B \subseteq A \subseteq k^*(B)$ and if B is semi-open set then A is also a semi-open set in (X, I, k^*).

Proof. Let B be semi-open set in (X, I, k^*) so we have $B \subseteq k^*(int^*(B)) \subseteq k^*(int^*(A))$. Thus $k^*(B) \subseteq k^*(int^*(A))$ implies $A \subseteq k^*(B) \subseteq k^*(int^*(A))$. We get $A \subseteq k^*(int^*(A))$. Hence A is semi-open set in (X, I, k^*).

Theorem 3.13. Let $A \subseteq Y \subseteq X$, where (X, I, k^*) is ideal closure space and (Y, I, k^*_Y) is a subspace of (X, I, k^*). Let A be semi-open set in (X, I, k^*) then A is semi-open set in (Y, I, k^*_Y).

Proof. Let A be semi-open set in (X, I, k^*) then
$A \subseteq k^*(int^*(A))$,
$\Longrightarrow A \cap Y \subseteq k^*(int^*(A)) \cap Y$
$\Longrightarrow A \subseteq k^*_Y(int^*_Y(A))$ [By the definition of subspace of ideal closure space]
Therefore A is semi-open in (Y, I, k^*_Y).

4. Semi Separation Axioms in Ideal Closure Space

In this section, we newly introduced semi separation axioms in ideal closure space. We already defined lower and higher separation axioms using closed and open sets in ideal closure space. Here we extended the separation axioms using semi open and semi closed sets namely semi separation axioms.

Definition 4.1. An ideal closure space (X, I, k^*) is said to be semi-T_0-space if and only if for every distinct points $x \neq y$ and $x \notin k^*(int^*(k^*(y)))$ or $y \notin k^*(int^*(k^*(x)))$.

Example 4.2. $X = \{a, b, c\}$, $\Im = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$, $I = \{\emptyset, \{a\}\}$.
$k^*(a) = \{a\}$; $k^*(b) = \{a, b\}$; $k^*(c) = \{a, c\}$; $k^*\{a, b\} = \{a, b\}$; $k^*\{b, c\} = X$; $k^*\{c, a\} = \{a, c\}$; $k^*(X) = X$; $k^*(\emptyset) = \emptyset$.
Let $a, b \in X$. Then there is a $k^*(int^*(k^*(a))) = \emptyset$ and $k^*(int^*(k^*(b))) = \{b, a\}$ such that $a \in k^*(int^*(k^*(b)))$, $b \notin k^*(int^*(k^*(a)))$
Let $b, c \in X$. Then there is a $k^*(int^*(k^*(b))) = \{b, a\}$ and $k^*(int^*(k^*(c))) = \{a, c\}$ such that $b \notin k^*(int^*(k^*(c)))$, $c \notin k^*(int^*(k^*(b)))$

Let $c, a \in X$. Then there is a $k^*(int^*(k^*(c))) = \{a, c\}$ and $k^*(int^*(k^*(a))) = \emptyset$ such that $c \notin k^*(int^*(k^*(a)))$, $a \in k^*(int^*(k^*(c)))$
Therefore (X, I, k^*) is semi-T_0-space.

Theorem 4.3. An ideal closure subspace of a semi-T_0-space is semi-T_0-space.

Proof. Let (X, I, k^*) be an ideal closure semi-T_0-space and (Y, I, k^*_Y) be the subspace of (X, I, k^*). Let x, y be two distinct points in (Y, I, k^*_Y). Since $(Y, I, k^*_Y) \subseteq (X, I, k^*)$ then either $x \notin k^*(int^*(k^*(y)))$ or $y \notin k^*(int^*(k^*(x)))$ implies that either $x \notin k^*(int^*(k^*(y))) \cap Y$ or $y \notin k^*(int^*(k^*(x))) \cap Y$. Hence (Y, I, k^*_Y) is a semi-T_0-space.

Definition 4.4. An ideal closure space (X, I, k^*) is said to be semi-T_1-space if and only if for every distinct points $x \neq y$ and $x \notin k^*(int^*(k^*(y)))$ and $y \notin k^*(int^*(k^*(x)))$.

Example 4.5. $X = \{a, b\}$, $\mathfrak{I} = \{X, \emptyset, \{b\}, \{a\}\}$, $I = \{\emptyset, \{b\}\}$.
$k^*(a) = \{a\}$; $k^*(b) = \{b\}$; $k^*(X) = X$; $k^*(\emptyset) = \emptyset$.
Let $a, b \in X$. Then there is a $k^*(int^*(k^*(a))) = \{a\}$ and $k^*(int^*(k^*(b))) = \{b\}$ such that $a \notin k^*(int^*(k^*(b)))$ and $b \notin k^*(int^*(k^*(a)))$
Therefore (X, I, k^*) is semi-T_1-space.

Theorem 4.6. An ideal closure subspace of a semi T_1-space is semi-T_1.

Proof. Let (X, I, k^*) be an ideal closure semi-T_0-space and (Y, I, k^*_Y) be the subspace of (X, I, k^*). Let x, y be two distinct points in (Y, I, k^*_Y). Since $(Y, I, k^*_Y) \subseteq (X, I, k^*)$ then there exist $x \notin k^*(int^*(k^*(y)))$ or $y \notin k^*(int^*(k^*(x)))$. This implies $x \notin k^*(int^*(k^*(y))) \cap Y$ and $y \notin k^*(int^*(k^*(x))) \cap Y$. Hence (Y, I, k^*_Y) is a semi-T_1-space.

Definition 4.7. An ideal closure space (X, I, k^*) is said to be semi-T_2-space if every distinct points $x \neq y$ and there exists disjoint semi-open sets G and H such that $x \in G$ and $y \in H$.

Example 4.8. $X = \{a, b, c\}$, $\mathfrak{I} = \{X, \emptyset, \{b\}, \{a\}, \{a, b\}\}$, $I = \{\emptyset, \{a, b\}\}$.
Ideal closure space (X, I, k^*) is defined by $k^*(a) = \{a, c\}$; $k^*(b) = \{b, c\}$; $k^*(c) = \{c\}$; $k^*\{a, b\} = \{a, b\}$; $k^*\{b, c\} = \{b, c\}$; $k^*\{c, a\} = \{a, c\}$; $k^*(X) = X$; $k^*(\emptyset) = \emptyset$.
Semi-open sets are $X, \emptyset, \{a, b\}, \{b, c\}, \{c, a\}, \{a\}, \{b\}, \{c\}$.
Let $a, b \in X$. Then there is a semi-open set $U = \{a\}$ and $V = \{b, c\}$ such that $a \in U$, $b \in V$ and $U \cap V = \emptyset$
Let $b, c \in X$. Then there is a semi-open set $U = \{b\}$ and $V = \{c, a\}$ such that $b \in U$, $c \in V$ and $U \cap V = \emptyset$
Let $c, a \in X$. Then there is a semi-open set $U = \{c\}$ and $V = \{a, b\}$ such that $c \in U$, $a \in V$ and $U \cap V = \emptyset$
Then (X, I, k^*) is semi-T_2-space.

Theorem 4.9. An ideal closure subspace of a semi-T_2-space is semi-T_2-space.

Proof. Let (X, I, k^*) be an ideal closure semi-T_2-space and (Y, I, k^*_Y) be the subspace of (X, I, k^*). Let x and y are two distinct points and there exists semi-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. Now $U \cap Y$ and $V \cap Y$ are semi-open sets in (Y, I, k^*_Y) such that $x \in U \cap Y$, $y \in V \cap Y$. Consider

$$\begin{aligned}(U \cap Y) \cap (V \cap Y) &= (U \cap V) \cap Y \\ &= \emptyset \cap Y \\ &= \emptyset.\end{aligned}$$

Therefore (Y, I, k^*_Y) is semi-T_2-space.

5. Some relationship between two classes of separation axioms and semi separation axioms in ideal closure space

In this section, we discuss the relationship between ordinary separation axioms and semi separation axioms in ideal closure space. Here we analyse some comparative results like every T_2-space is semi-T_2-space, every T_1-space is semi-T_1-space etc.,

Theorem 5.1. Every T_2-space is semi-T_2-space.

Proof. Let (X, I, k^*) be T_2-space and let x, y ∈ X, x ≠ y, then there exist U and V open sets in X such that x ∈ U and y ∈ V, U∩V = ∅. Since every open set is semi-open set. Then U and V are semi-open sets in X such that x ∈ U and y ∈ V, U∩V = ∅. Hence (X, I, k^*) is semi-T_2-space.

Theorem 5.2. Every T_1-space is semi-T_1-space.

Proof. Let (X, I, k^*) be ideal closure T_1-space and let $x, y \in$ X, x ≠ y, then $x \notin k^*(y)$ and $y \notin k^*(x)$. Using the result 3.8, we have $k^*[int^*(k^*(A))] \subseteq k^*[A]$ for every semi-closed set $A \subset X$. So $x \notin k^*(y)$ implies that $x \notin k^*(int^*(k^*(y)))$ and $y \notin k^*(x)$ implies that $y \notin k^*(int^*(k^*(x)))$. Therefore every T_1-space is semi-T_1-space.

Theorem 5.3. Every T_0-space is semi-T_0-space.

Proof. The way of the proof is similar to the proof of theorem 5.2

Theorem 5.4. Every semi-T_2-space is semi-T_1-space.

Proof. Let (X, I, k^*) be ideal closure semi-T_2-space and let x, y ∈ X, x ≠ y, then there exist U and V semi-open sets in X such that x ∈ U and y ∈ V, U∩V = ∅. Then $X - U$ and $X - V$ are semi-closed sets in X and let $X - U = A$, $X - V = B$ such that $x \in X - V$ and $y \in X - U$ such that $A \cap B = \emptyset$.
Since taking k^*-operator on both sides we get,
$\implies A \cap B = \emptyset$
$\implies k^*(A \cap B) = k^*(\emptyset)$
$\implies k^*(A) \cap k^*(B) = \emptyset$
Here $k^*(A)$ and $k^*(B)$ are distinct. We have $k^*[int^*(k^*(A))] \subseteq k^*[A]$, $k^*[int^*(k^*(B))] \subseteq k^*[B]$. This implies $k^*(int^*(k^*(A)))$ and $k^*(int^*(k^*(B)))$ also distinct.
This implies $x \in k^*(int^*(k^*(B)))$, $y \notin k^*(int^*(k^*(B)))$ and $y \in k^*(int^*(k^*(A)))$, $x \notin k^*(int^*(k^*(A)))$. Hence (X, I, k^*) is semi-T_1-space.

Theorem 5.5. Every semi-T_1-space is semi-T_0-space.

Proof. Let (X, I, k^*) be a semi-T_1-space. Let $x, y \in X$, $x \neq y$, from the definition of semi-$T1$-space both $x \in k^*(int^*(k^*(x)))$, $y \notin k^*(int^*(k^*(x)))$ and $y \in k^*(int^*(k^*(y)))$, $x \notin k^*(int^*(k^*(y)))$. From the rule of addition either $x \in k^*(int^* k^*(x))$, $y \notin k^*(int^*(k^*(x)))$ or $y \in k^*(int^*(k^*(y)))$, $x \notin k^*(int^*(k^*(y)))$, which precisely the definition of a semi-T_0-space.

Theorem 5.6. Every semi-T_2-space is semi-T_0-space.

Proof. The way of the proof is similar to the proof of the theorem 5.4

Result 5.7. *From the above result, we conclude that the relation between two classes of separation axioms and semi-separation axioms in ideal closure space are given below.*

$$\begin{array}{ccc} T_2\text{-space} & \Rightarrow & \text{semi-}T_2\text{-space} \\ \Downarrow & & \Downarrow \\ T_1\text{-space} & \Rightarrow & \text{semi-}T_1\text{-space} \\ \Downarrow & & \Downarrow \\ T_0\text{-space} & \Rightarrow & \text{semi-}T_0\text{-space} \end{array}$$

6. Conclusion

In this paper, basic concepts of semi separation axioms in ideal closure space is introduced. Also the relation between separation properties and semi separation properties in ideal closure space (X, I, k^*) are derived. In addition, properties of semi-open and semi-closed sets, union and intersection of semi-open sets are discussed.

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